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IGNITION DELAY TIME MODULATION AS A CONTRIBUTION TO THERMO-ACOUSTIC INSTABILITY IN SEQUENTIAL COMBUSTION

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ABSTRACT

Pressure pulsations due to combustion instabilities have been encountered in a premixed sequential gas turbine combustor. Measured noise spectra display one or several distinct peaks at Strouhal numbers significantly larger than unity. Height and location of the peaks depend in a sensitve manner on fuel type and/or operating conditions.

The paper identifies a possible mechanism of the observed combustion instability and presents a mathematical model of acoustic self-excitation. The mechanism of self-excitation comprises interactions between the acoustic field in the fuel injector / burner with the ignition delay time of the fuel-air mixture and the heat release intensity:

- pressure drop in the fuel injector nozzle changes with variations of the acoustic pressure in the burner,
- variations of pressure drop and air flow velocity modulate the fuel concentration,
- acoustic perturbations in the pre-flame region influence the delay time for self-ignition and consequently lead to fluctuations of flame velocity and -position.
- fluctuations of flame velocity influence the refracation of acoustic waves at the flame front.
- fuel inhomogeneities modulate the heat release rate and consequently the rate of volume production by the flame.

Based on this structure of a self-excitation mechanism, an analytical model has been developed and used to compute eigenfrequencies and growth rates of instabilities. Some characteristics of the suggested self-excitated instabilities as they are predicted by the model match well with empirical information.

INTRODUCTION

Thermoacoustic instabilities are a cause for concern in combustion applications as diverse as domestic heating, gas turbines or rocket engines. Combustion instabilities can not only increase emissions of noise or pollutants such as unburnt hydrocarbons or oxides of nitrogen, but also lead to very high levels of pressure pulsations, resulting in structural damage of the combustor. For stationary gas turbines, the drive for lower emissions of oxides of nitrogen has lead to the wide-spread introduction of premix burners and convectively cooled combustion chambers during the last decade, resulting in reduced flame stability or flame "anchoring" and lower acoustic damping, respectively. Consequently, the awareness of thermo-acoustic combustion phenomena in gas turbine combustors has increased with the introduction of these technologies, and so has the understanding of thermo-acoustic instabilities in premixed combustion systems.

Several possible mechanisms of instability have been identified, involving e.g. flame front kinematics (see Krüger et al. (1999) or Hubbard and Dowling (1998), Dowling (1999) and references therein), coherent fluid dynamic structures initiated by shear flow instabilities (Paschereit et al., 1998), so-called entropy waves (Humphrey and Culick (1987), Keller (1995), Polifke et al. (1998)) or random sources due to turbulent processes driving an acoustic eigenmode of combustor or supply into resonance. A mechanism of particular importance for lean conditions may be self-excited feedback loops involving fuel inhomogeneities caused by acoustic perturbations in the vicinity of the fuel injector. This mechanism has already been discussed in the classic work by Putnam (1971), and has enjoyed renewed attention in the recent past, see e.g. Dowling (1999), Keller (1995), Lieuwen et al. (1998, 1999), Polifke et al. (1998), Schuermans et al. (1999) or Straub and Richards (1998). Typically, time lags due to convection, finite chemical rates or flame front kinematics play an important role by bringing fluctuations of heat release into phase with pressure oscillations, thereby satisfying the fundamental Rayleigh criterion.

When combustion instabilities occur in an installation, the quality and quantity of empirical data is often not sufficient to identify unambiguously the instability mechanism which is to be held responsible. This is particularly so if the instability occurs not in a test rig, heavily instrumented and perhaps with optical access, but in a field installation. In this case, it is often difficult to obtain even the most fundamental data, such as reliable spectra of pressure pulsations. If experimental data are scarce, the engineer is challenged to identify likely candidates for the root mechanism of the instability, and it is often essential that analytical or numerical studies provide additional information. Eventually, this kind of deductive reasoning should shorten the list of candidates to only a few entries and - more importantly - suggest effective measures to suppress the instability. The present paper makes a contribution in this regard.

Combustion instabilities in a sequential combustor (Joos et al., 1997) are scrutinized. The instabilites observed display one or several distinct peaks in the power spectrum of pressure oscillations. Height and location of the peaks respond in a very sensitive manner to changes in operating conditions, in particular combustor inlet temperature, fuel type and details of fuel injector and burner geometry.

We propose a mechanism of the observed combustion instability and present a mathematical model of acoustic self-excitation. The mechanism of self-excitation is due to feedback from the acoustic field in the combustor with the fuel injector and *the ignition delay time* of the fuel-air mixture. The ignition delay time - characterisitic of the self-ignited mode of combustion in the sequential combustion chamber - in turn controls the location of heat release. The structure of the feedback loop may be represented as follows:

- pressure drop in the fuel injector nozzle changes with variations of the acoustic pressure in the burner,
- variations of pressure drop and air flow velocity modulate the fuel concentration,
- acoustic perturbations in the pre-flame region influence the delay time for self-ignition and consequently lead to fluctuations of flame velocity and -position.
- fluctuations of flame velocity influence the refracation of acoustic waves at the flame front.
- fuel inhomogeneities modulate the heat release rate and consequently the rate of volume production by the flame.

According to this scheme, the acoustic field in the combustor, the fuel supply system and the chemical kinetics and kinematics of the self-ignition process constitute a system with time-delayed feedback, which under some conditions may become unstable. The time-delays are convective and chemical-kinetic in nature. Comparing our work with the studies of combustion instabilities cited above, we note that the treatement of the self-ignition condition and the explicit inclusion of flame speed fluctuations in the acoustic coupling conditions across the flame sheet are novel.

Based on the above structure of the self-excitation mechanism an analytical model has been developed and used to compute eigenfrequencies and growth rates of instabilities. Some characteristics of the suggested self-excited instabilities as they are predicted by the model match empirical information well. Note that in this paper, we do not explore alternative explanations for the observed instabilities, and we do not argue that the mechanism presented is indeed ultimately responsible for the phenomena observed. To do so confidently and convincingly would require more detailed experimental data than presently available.

NOMENCLATURE

A_{Burner}	burner cross sectional area
c	speed of sound
C^o	convective characteristic
e	specific energy
E	activation energy
F,G	coefficients
f	frequency
$J^{+/-}$	Riemann invariants
k	Arrhenius prefactor
L	the burner diameter
m_F	mass flow rates of fuel
m_A	mass flow rates of air
M	Mach number
p	pressure
q	mass flux
Q	heat release
s	entropy
S	flame front velocity
St	Strouhal number
t_l, t_f	time instants of the particle ignition and mixing
u	mean gas velocity
V	specific volume
x_f	coordinate of the flame front
x_l	coordinate of the injector
α	parameter
δf	perturbation of parameter f
Δp	the pressure drop
ϕ	equivalence ratio
γ	the ratio of specific heats.
ρ	density
τ	delay of ignition
ω	frequency of oscillations
ζ	pressure loss coefficient
subscripts 1,(2)	designate parameters ahead (behind) the flame
subscripts f,l	designate the parameters at the flame front and at the mixing zone

SEQUENTIAL COMBUSTION

In sequential gas turbine combustion, vitiated air from a primary combustor enters, after passing through a first turbine stage, a fuel injector / burner at elevated pressures and temperatures. Due to the high temperature of the fuel/air mixture, self-ignition occurs after a (short) ignition delay time. For further details on operating conditions and details of burner / fuel nozzle geometry we refer to Joos et al. (1997).

Concerning the thermo-acoustic characteristics of sequential combustion, an additional comment has to be made: The conservation equations for mass, momentum and energy across a flame suggest that the strength of the coupling between heat release fluctuations and acoustics is roughly proportional to the relative temperature increase T_h/T_c-1 across the heat release zone. The relative temperature increase is approximately 2 for a standard modern premixed gas turbine combustor. It cannot be much less than this for today's preheat temperatures due to lean extinction at insufficient flame temperature. Contrary, in a sequential combustor with its high inlet temperature, the relative temperature increase may be lower than 0.5. Therefore, a sequential combustor should exhibit a very low intrinsic susceptibility to thermo-acoustic combustion instabilites. Nevertheless, the present analysis shows that feedback mechanisms known from standard premixed combustors may also be active and cause instability in a sequential combustor.

BASICS OF MODEL FORMULATION

The following simplified, one-dimensional scheme of a premixer/burner in self-ignited mode of operation is proposed in order to explore some basic instability characteristics of the system fuel supply - acoustic field - chemical kinetics, as introduced above. Conceptually, the flow is to be devided into three main regions (see Fig.1):

- the *mixing region*, where the mixing of the fuel with the hot vitiated air takes place.
- the *induction* or *pre-flame region*, where the fuel hydrocarbon species decompose via chain branching reactions into intermediate free radicals ("*ignition precursors*") (see Spadaccini and Colket, (1994) until
- ignition occurs, leading to rapid depletion of the primary fuel and an increase in temperature.

Within each region the flow is assumed to be uniform. The flame front is assumed to be infinitesimally thin, resulting in a discontinuous jump of flow parameters across the region of heat release. Also, the burner cross-sectional area is assumed to be constant. This decomposition of the flow field represents a idealization of the real flow, in reality there are no sharp boundaries between the various regions due to turbulent dissipation and the complex three-dimensional structure of the flow and mixing fields involved. Also the burner cross-sectional area varies by approximately 20 % over its length. Given that we are interested in presenting the essential physical features of the instability mechanism proposed, the assumptions made seem appropriate.

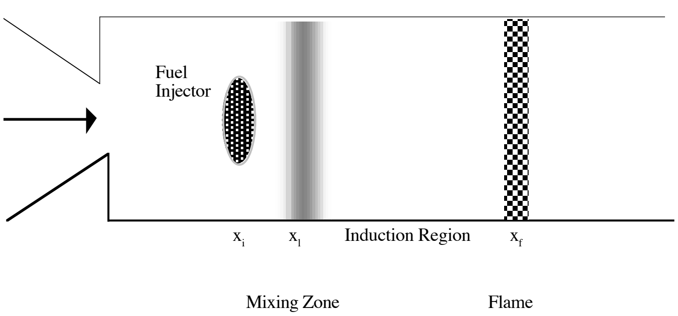

Fig. 1: Sketch of the idealized fuel injector / burner configuration investigated in this study.

Flow parameters change discontinuously at the flame front, but are coupled by the laws of conservation of mass, momentum and energy, see e.g. Landau & Lifshitz (1987). The flow parameters ahead and behind the flame front are marked below by subscripts "1" and "2", respectively:

$$\rho_1(u_1 - S) = \rho_2(u_2 - S) = q, \tag{1}$$

$$p_1 + qu_1 = p_2 + qu_2, \tag{2}$$

$$e_2 = e_1 + \frac{p_1 + p_2}{2}(V_1 - V_2) + Q\,. \tag{3}$$

Here $\rho, u, S, q, e, p, V, Q$ denote density, velocity, flame front velocity, momentum flux, specific energy, pressure, specific volume and heat release per unit mass, respectively.

To derive reflection and transmission conditions at the flame front, assume that perturbations of characteristic magnitude $\delta << 1$ are introduced in the originally steady flow. Variation of the conservation laws due to these perturbations (indicated by the δ symbol) lead to the following relationships:

$$\delta p_1 + \frac{\delta\rho_1}{\rho_1} qu_1 + (\rho_1 u_1 + q)\delta u_1 - \delta p_2 - \frac{\delta\rho_2}{\rho_2} qu_2 - (\rho_2 u_2 + q)\delta u_2 =$$

$$= (\rho_1 u_1 - \rho_2 u_2)\delta S\,. \tag{4}$$

$$\frac{\delta\rho_1}{\rho_1} q + \rho_1 \delta u_1 - \frac{\delta\rho_2}{\rho_2} q - \rho_2 \delta u_2 = (\rho_1 - \rho_2)\delta S, \tag{5}$$

$$A_2 \delta p_2 + B_2 \delta\rho_2 - A_1 \delta p_1 - B_1 \delta\rho_1 = 2(\gamma - 1)\delta Q\,, \tag{6}$$

where the coefficients *A, B* are defined as follows:

$$A_i = (\gamma + 1)V_i - (\gamma - 1)V_j,$$

$$B_i = -\frac{(\gamma + 1)p_i + (\gamma - 1)p_j}{\rho_i^2},$$

where $\{i, j\} = \{1,2\}$ or $\{i, j\} = \{2,1\}$. As usual, γ denotes the ratio of specific heats.

It is convenient to perform the further consideration in terms of the Riemann invariants. Perturbations of pressure and velocities are expressed via Riemann invariants by the formulae

$$\delta p(t,x) = \rho c \left\{ J^+\left(t - \frac{x}{u + c}\right) - J^-\left(t - \frac{x}{u - c}\right)\right\}, \tag{7}$$

$$\delta u(t,x) = \left\{ J^+\left(t - \frac{x}{u + c}\right) + J^-\left(t - \frac{x}{u - c}\right)\right\}. \tag{8}$$

Perturbations of densities are given by formulae

$$\delta\rho = \frac{\rho}{c}\left(J^+ - J^-\right) + \alpha\,\delta s\,,$$

$$\alpha = \left(\frac{\partial \rho}{\partial s}\right)_p = -\frac{\gamma - 1}{\gamma}\frac{\rho^2 T}{p} \ .$$

Boundary conditions at the inlet and the outlet of the burner must be added to the system of equations (4)-(8) to complete the mathematical formulation of the problem. An accurate description of the boundary conditions requires consideration of the interaction of the flow in the burner with the flow in the regions adjacent to the burner, and represents a very complex problem itself. We shall employ boundary conditions based on the following simplifying assumptions: Since the flow is subsonic, two boundary conditions are needed, one at the inlet and one at the outlet of the burner. The crossection of the flow changes at the inlet and the outlet discontinuously. Since the jumps in cross-sectional area are large and the Mach number is small, we may assume that the velocity is fixed at the inlet and the pressure at the outlet. In terms of Riemann invariants the boundary condition at the inlet is

$$J_1^+(t,0) + J_1^-(t,0) = 0 \ ,. \tag{10}$$

At the outlet of the burner/ combustor model, the pressure node implies:

$$J_2^+(t,L) - J_2^-(t,L) = 0 \tag{11}$$

Concerning the boundary condition δs_1 for entropy at the inlet of the burner, we note that in a sequential combustor it is quite possible that $\delta s_1 \neq 0$ due to entropy disturbances generated in the first combustion chamber. Such fluctuations can be treated without essential difficulty with the methods employed in this work. Nevertheless, in order to present our main ideas more clearly, we shall assume that fluctuations of entropy are absent.

We recognize that the formulated boundary conditions (10) and (11) are simplified and may cause objections because they do not account for losses. However, the main goal of the model is to quantify the excitation mechanisms but not the damping. As it will be shown below, the frequencies of the pressure waves in the burner calculated with these boundary conditions are in excellent agreement with the experiment. It means that the boundary conditions chosen provide the right phase shift due to reflection.

Note that with δS appearing in Eqs. (4) and (5), we employ a kinematic formulation, where it is taken into account explicitly that changes in flame speed will influence the refraction of acoustic waves at the flame. In order to close the system of equations representing our thermo-acoustic model, it is necessary to express both δS and δQ in terms of known operating parameters, variables pertaining to the mean flow and acoustic variables. The required closure relations are provided in the next two Sections.

CLOSURE FOR FLUCTUATIONS OF FLAME POSITION DUE TO PERTURBATION OF SELF IGNITION CONDITION

Consider the perturbations of a definite frequency ω . The Riemann invariants are represented as

$$J_i^\pm(t,x) = J_i^\pm \exp\left[i\omega\left(t - \frac{x}{u_i \pm c_i}\right)\right]. \tag{12}$$

Perturbations of convected quanties, e.g. entropy, have the form

$$s_i(t,x) = s_i \exp\left[i\omega\left(t - \frac{x}{u_i}\right)\right] \ .$$

Perturbations of the flame front velocity due to acoustic disturbances will be determined on the basis of a simple, global, chemical kinetic model of the self-ignition process characteristic of sequential combustion. Both convective and chemical-kinetic effects on the self-ignition process shall be considered. Assume that the self-ignition occurs after the life time of the moving fuel "particle" reaches the time of the ignition delay:

$$\int_{t_l}^{t_f} \frac{dt}{\tau} = 1 \ . \tag{13}$$

Here t_f , t_l are time instants of the particle ignition and mixing, while τ is the value of the delay of ignition, which may in general depend instantaneously on the local physico-chemical conditions encountered by the particle. Along the convective characteristic (see Fig. 2), the increment of time may be expressed via the increment of coordinate as

$$dt = \frac{dx}{u} \ .$$

The self-ignition condition rewritten in terms of the spatial coordinate is then

$$\int_{x_l}^{x_f} \frac{dx}{\tau u} = 1 \ . \tag{14}$$

Here x_f, x_l are the spatial coordinates of the flame front (the end of the induction region) and the mixing plane, respectively.

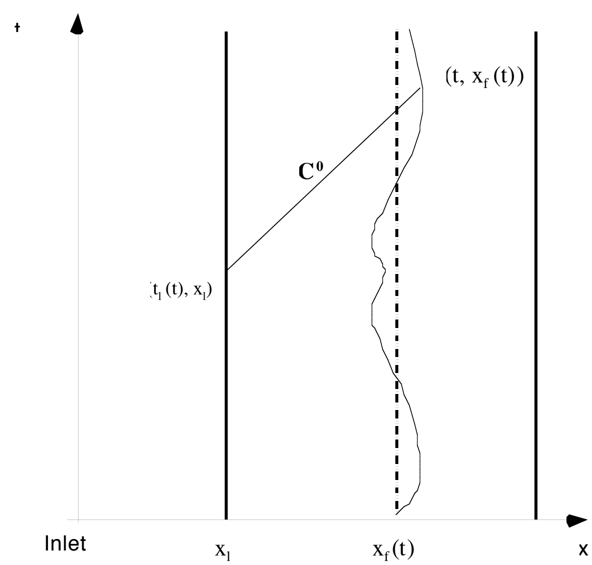

Fig. 2 The fluctuating flame and the convective characteristic C^0 in the *x-t* diagram.

If a fuel particle's self-ignition time τ is changed due to perturbations of the steady state along the particle's path, the position of the flame will no longer be constant. The rate of change of flame position is equal to the flame speed's deviation from its mean value (see also Fig. 2):

$$\delta S = \frac{dx_f(t)}{dt}. \qquad (15)$$

Differentiating the self-ignition condition (14) with respect to t yields

$$\frac{\delta S}{\tau u} + \int_{x_l}^{x_f} \frac{d}{dt}\left(\frac{1}{\tau u}\right) dx = 0.$$

Because all terms which are *0-th* order in δ are constant in space and time, the above expression can be re-written as follows:

$$\delta S = \frac{1}{u}\int_{x_l}^{x_f} \frac{d\ \delta u}{dt} dx + \frac{1}{\tau}\int_{x_l}^{x_f} \frac{d\ \delta\tau}{dt} dx \qquad (16)$$

This expression provides the required closure relation for the fluctuations of flame speed δS. The first term on the r.h.s of the above equation expresses the fact that acoustic velocity fluctuations along a fuel particle's trajectory will influence its movement and thereby also the flame front position. Similarly, the second term describes how acoustic perturbations of pressure and temperature seen by the particle as it moves through the mixing section, will modify the instant and the location of self-ignition. Integrating over the fuel particle's history allows to determine the accumulated effects of acoustic perturbations on the self-ignition condition. Details are given in Appendix A, here we present only the results:

$$\frac{1}{u}\int_{x_l}^{x_f} \frac{d\ \delta u}{dt} dx = \left((M+1)J^+(t',x') - (M-1)J^-(t',x')\right)\Big|_{x_l}^{x_f} \qquad (17)$$

for the velocity term and

$$\frac{1}{\tau}\int_{x_l}^{x_f} \frac{d\ \delta\tau}{dt} dx = -(\gamma-1)\frac{E}{T}M(M+1)\left(J^+(t',x') + J^-(t',x')\right)\Big|_{x_l}^{x_f} \qquad (18)$$

for the chemical-kinetic effect. Note that the parameter E is an activation energy, where we have followed Spadaccini and Colket (1994) and assumed a global Arrenius kinetics for the self-ignition process:

$$\tau = k \exp\left(\frac{E}{T}\right) \cdot \qquad (19)$$

The simplified flame model underlying the derivation of the closure relations (17) and (18) in Appendix A certainly does not describe the flame structure in the combustor, which must be expected to be very complex. However, the model seems to predict reasonably well the response of the flame front velocity to the flow perturbations, and reflection coefficients of the pressure waves from the flame. The description of these phenomena, as given above, is based in fact only on the assumptions of an infinitesimally thin flame and a quasi-stationary response to pressure waves, certainly correct for the frequencies of interest.

CLOSURE FOR HEAT RELEASE RATE FLUCTUATIONS

As previously noted by many authors (see the references given in the Introduction, in particular the illuminating discussion in (Dowling, 1999)), the heat release term in Eqn. (6) will respond to fluctuations in momentary fuel concentration at the flame. This fuel concentration in turn is determined by acoustic fluctuations at the location of fuel injection and by a convective time flag from fuel injection to flame

$$\Delta t = \frac{x_f - x_i}{u_1}, \qquad (20)$$

where x_f, x_i are coordinates of the flame front and fuel injection plane, respectively (See also Figure 1). In the present work, these effects are described by an expression of the following form:

$$\delta Q(t, x_f) = F\ J_1^+\left(t - \Delta t - \frac{x_f}{u_1 + c_1}\right) + G\ J_1^-\left(t - \Delta t - \frac{x_f}{c_1 - u_1}\right), \qquad (21)$$

Among other factors, the pressure drop Δp across the fuel injector nozzle determines the coefficients F and G. For details, the interested reader is referred to Appendix B.

PRESENTATION OF NUMERICAL RESULTS

If the expressions for the heat release (21) and the flame front velocity perturbation Eqs. (17-19) are substituted into the flame front conditions (4)-(6), and the Riemann invariants J_1^+, J_2^- are expressed via J_1^-, J_2^+ from the boundary conditions (10), (11), we come to a system of three linear equations relative three unknown constants J_1^-, J_2^+, s_2

$$\vec{A}^-\ J_1^- + \vec{B}^+\ J_2^+ + \vec{S} s_2 = 0$$

The system of equations has nontrivial solutions if its determinant is equal to zero. The determinant is a function of the frequency :

$$\det(\omega) = 0 \qquad (22)$$

The solutions of this transcendental equation give the

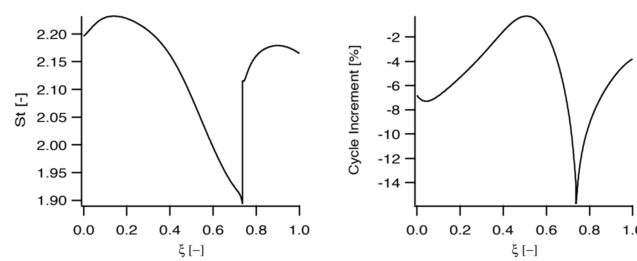

Fig. 3 Strouhal number (left) and cycle increment (right) as function of normalized flame position ξ of first mode with *E = 20000* K and fuel injector pressure loss Δ*p* = 10 bar. For the combination of parameters shown, the first mode is stable for all flame positions considered.

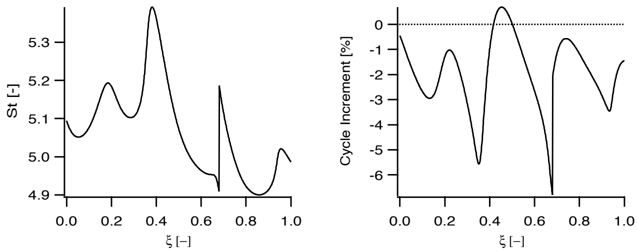

Fig. 4 Strouhal number (left) and cycle increment (right) as function of normalized flame position of second mode (St ~ 5). A (weak) instability is observed for $\xi \approx 0.4$.

eigenfrequencies ω_n of the system. If the imaginary part of an eigenfrequency is positive

$$\mathrm{Im}(\omega_n) > 0 \ ,$$

the corresponding mode will decay with time. However, if the imaginary part of any eigenfrequency is negative, infinitesimal excitation of the corresponding mode will grow with time exponentially and the system is unstable.

The transcendental equation (22) has to be solved numerically. Input data such as length of the burner / combustor, pressure drop, the fuel and air mass flow rates and other hydrodynamic parameters (velocity, pressure, density, temperature) have been chosen to resemble machine conditions. Because we are interested in exploring in a rather qualitative manner some characteristic features of the observed instabilities, a more precise modelling of actual machine conditions and geometrical detail is not required for the present presentation.

The (steady state) location of the flame front x_f is determined by flow speed, mixing processes, chemical kinetics and so on. It has been noted that changes in operating conditions which are likely to change the location in heat release often also influence combustor stability strongly. Therefore, the location of the flame front x_f was varied over a certain range Δx as the independent variable for these plots. On Figs. 3, 4 and 5 the dependence of the first three eigenfrequencies on the distance between the fuel injection plane and flame front are shown. All quantities are non-dimensionalized, i.e. the real part of the eigenfrequencies $\omega_n, (n = 1, 2, 3)$ is plotted as Strouhal number St, while the imaginary part is plotted as a *cycle increment*. Note that instead

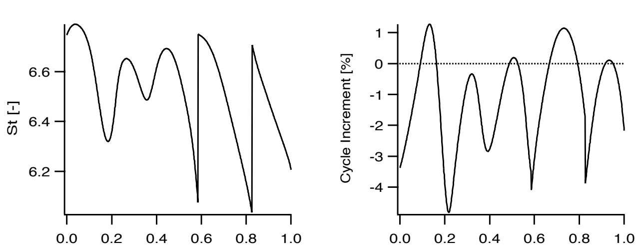

Fig. 5 Strouhal number (left) and cycle increment (right) as function of normalized flame position of third mode (St ~ 6.5). Strong instabilities found for $\xi \approx 0.1$ and $\xi \approx 0.7$)

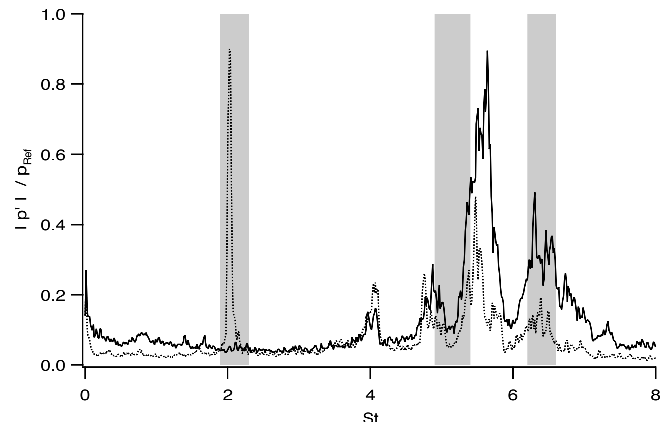

Fig. 6 Power spectrum distribution of pressure fluctuations in gas turbine combustor vs. Strouhal number. Data are from two different operating conditions. Regions where -- according to the analysis in this paper -- instabilities may occur are indicated by the gray areas.

of a normalized frequency, a Strouhal number St has been chosen for presentation of the results, because this is the natural choice for instability mechanisms involving hydrodynamic instabilities, which have also been explored as possible causes for the observed pressure pulsations. The cycle increment equals the percentage by which the amplitude of an infinitesimal perturbation grows during one cycle of the oscillation; negative cycle increments indicate stability. The flame front location is also given in non-dimensional form as $\xi = x / \Delta x$. The margins of variation of this parameter have been chosen in calculations to sweep the space between the fuel lance and the outlet of the burner.

For the calculations presented here, we have chosen an activation energy E = 20000 K, which is according to Spaddacini and Colket (1994) the right order of magnitude if natural gas is used as fuel. For oil operation, a somewhat higher activation energy, perhaps E = 25000 K would be appropriate (Spaddaccini and TeVelde, 1982). We note that the calculations have readily produced eigenmodes with Strouhal numbers in the same range as observed in experiment, i.e. St = 2 - 7, and indeed even quite close in value to the location of power spectrum peaks observed in machine operation (see Fig. 6.). For the parameters chosen here, only the second and third mode exhibit "dangerous" regions of the flame front with positive cycle increment.

The model appears to be very sensitive to the parameters entering the closure relations, Eqs. (17), (19) and (21). For example, reduction of the fuel lance pressure drop Δp by 10 – 20 % makes the first mode unstable (not shown). Similarly, increasing the value of the activation energy E tends to promote instability. The instability conditions are, as it turns out, particularly sensitive to the location of the flame front and the fuel injection plane.

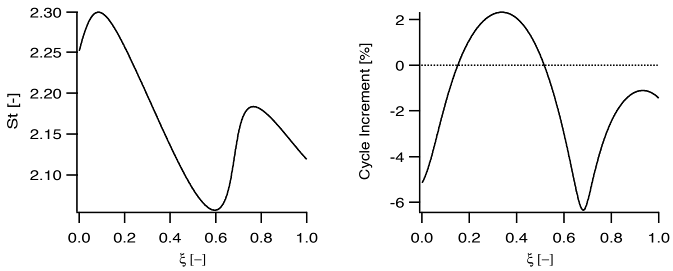

Fig. 7 Strouhal number (left) and cycle increment (right) as function of normalized flame position of first mode with fixed flame ($\delta S = 0$). Strong instabilities found for normalized flame position $\xi \approx 0.3$

To check whether the inclusion of the velocity δS does have an effect on the stability of the model system, we have also solved the system with δS set to zero, and we could confirm that the flame velocity fluctuations in general do have a significant influence. One such result is shown in Fig. 8. Comparing with Fig. 4, we note that in this case the inclusion of δS has actually increased stability of the model. It may seem puzzling that the inclusion of an additional instability mechanism can have a stabilizing influence. However, we observe again (see also Polifke et al, 1998) that two (or more) self-exciting feedback loops can interfere destructively as well as constructively to promote or reduce stability.

CONCLUSIONS

A mechanism of self-excited combustion instability in a sequential combustor involving coupling between acoustics and self-ignition time, flame velocity, fuel injection, and heat release intensity has been presented. A simple numerical model based on this mechanism was able to reproduce several characteristic features of combustion instabilities observed in machine operation:

- a strong sensitivity of combustor stability on all factors which influence flame position. Among these are preheat temperature, fuel concentration and type, fuel injector geometry an pressure drop, the presence of flame holders, etc.
- the occurrence of one or several distinct peaks at rather high frequencies (Strouhal numbers in the range St = 2.5 - 7, where St = fL/U is based on mean flow speed and burner diameter.

The mechanism proposed suggests - and this has been confirmed in numerical studies - that any measures that spread out the distribution of convective or chemical-kinetic time lags in the axial direction should in general improve combustion stability. The empirical information available suggests that this is indeed the case, although - and here we refer to the discussion in the Introduction - the available data is not detailed enough to rule out other explanations for the phenomena observed.

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APPENDIX A: INTEGRATION OVER FUEL PARTICLE HISTORIES

Because it is necessary to compute the effect of perturbations on the self-ignition condition accumulated over the particle history, the integrals in Eq. (16) from x_I to x_f must be interpreted as integrals along the convective characteristic C^0, see Fig. 2. For a given particle, which self-ignites at time t and position $x_f(t)$, one must "look backward" in time and space along the characteristic. Fortunately, because we are only considering terms first order in δ, it is not necessary to consider the effects of perturbations on the characteristic itself, i.e. we may assumed that the particle has moved along the mean characteristic described by $dx = u\,dt$.

Consider for the moment only the first term on the right hand side, and introduce single quotes to indicate points (t', x') on the characteristic C^0:

$$\frac{1}{u}\int_{x_I}^{x_f}\frac{d}{dt}\frac{\delta u}{}\,dx = \frac{1}{u}\int_{x_I}^{x_f}\frac{d}{dt}\left(J^+(t',x') + J^-(t',x')\right)dx'.$$

To evaluate the integral, it is necessary to explicitely account for implicit dependencies of t' on x', *i.e.* to express the arguments of the Riemann invariants as functions of the independent variable t, the variable of integration x', and known (constant) quantities like x_f, u, etc. Obviously, along C^0

$$t' = t - \frac{x_f - x'}{u}, \tag{A1}$$

which allows to re-write the argument of the Riemann invariants in the desired form:

$$t' - \frac{x'}{u \pm c} = t - \frac{x_f}{u} \pm \frac{1}{M \pm 1}\frac{x'}{u}. \tag{A2}$$

It follows that

$$\frac{d}{dt}J^{\pm}(t',x') = i\omega\, J^{\pm}(t',x')$$

while

$$\int_{x_I}^{x_f} J^{\pm}(t',x')\,dx' = \pm\frac{u(M \pm 1)}{i\omega}J^{\pm}(t',x')\Big|_{x_f}^{x_f}. \tag{A3}$$

Combining these results, one obtains for the contribution of velocity fluctuations along the characteristics to fluctuations in the position of the self-ignited flame the following expression:

$$\frac{1}{u}\int_{x_I}^{x_f}\frac{d\,\delta u}{dt}dx = \left((M+1)J^+(t',x') - (M-1)J^-(t',x')\right)\Big|_{x_I}^{x_f}. \tag{A4}$$

Now we turn to the second term on the r.h.s. of (18). In order to estimate how changes in the ignition delay due to acoustic perturbations integrate along the characteristic, we follow Spadaccini and Colket (1994) who assumed a global Arrenius kinetics for the self-ignition process

$$\tau = k\exp\left(\frac{E}{T}\right). \tag{A5}$$

Note that there is no dependence of the ignition delay of fuel concentrations, which is in accord with Spaddaccini and Colket's (1994) findings. Furthermore, we have lumped the influence of oxygen concentration on ignition delay into the prefactor k. This is justified because the relative magnitude of fluctuations of oxygen concentrations is small, and so is the effect of such fluctuations on flame position.

With these assumption, and since the flow ahead of the flame front is assumed to be isentropic, the time of ignition delay is a function of pressure only, and deviations from the mean value are proportional to pressure fluctuations:

$$\delta\tau = \frac{\partial\tau}{\partial p}\delta p.$$

For an ideal gas, the derivative of ignition delay with respect to pressure is

$$\frac{\partial\tau}{\partial p} = -k\exp\left(\frac{E}{T}\right)\frac{E}{T^2}\frac{\partial T}{\partial p} = -\frac{\gamma - 1}{\gamma}\frac{E}{T}\frac{\tau}{p}. \tag{A6}$$

Substituting this expression into the second term on the r.h.s of Eqn. (20) for the flame velocity perturbation, replacing δp by the appropriate combination of Riemann invariants, taking the derivative with respect to t and integrating over x' (while again making sure to take into account the dependence of t' on x' as above), we obtain eventually:

$$\frac{1}{\tau}\int_{x_I}^{x_f}\frac{d\,\delta\tau}{dt}dx = -(\gamma - 1)\frac{E}{T}M(M+1)\left(J^+(t',x') + J^-(t',x')\right)\Big|_{x_I}^{x_f}. \tag{A7}$$

Note that when evaluating (A4) and (A7), the arguments of the Riemann invariants at the endpoints x_I and x_f of the induction region must again be understood to be the endpoints of the characteristic C^0, see Eqn. (B1)

APPENDIX B: FLUCTUATION OF HEAT RELEASE RATE DUE TO PERTURBATION OF FUEL CONCENTRATION

As previously noted by many authors (see the references given in the Introduction, in particular the illuminating discussion given by Dowling (1999)), the heat release term δQ in Eqn. (6) will respond to fluctuations in momentary fuel concentration or equivalence ratio $\delta\phi$

$$\frac{\delta Q}{Q} \sim \frac{\delta\phi}{\phi}. \tag{B1}$$

To work out the dependence of fuel equivalence fluctuations on burner acoustics, consider a fuel injector system as sketched in Fig. (3). Fuel is supplied from a plenum, where constant pressure is assumed, through a small nozzle into the premixing section. In the quasi-stationary limit the pressure drop $\Delta p = p_{Plenum} - p$ across the nozzle and its pressure loss coefficient ζ determine the fuel velocity

$$\Delta p = \frac{\zeta}{2}\rho u_F^{\,2}. \tag{B2}$$

Changes in the mass flow rates of fuel $\dot{m}_F$ and air $\dot{m}_A$ will affect the equivalence ratio as follows:

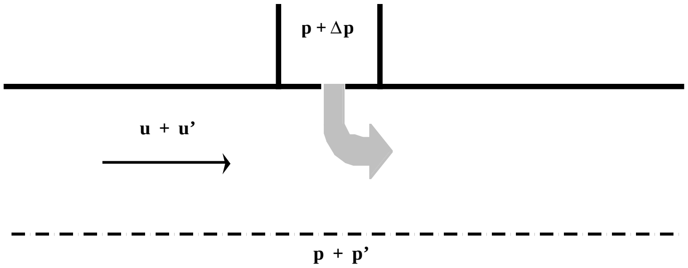

Fig. 8 Sketch of fuel injector with pressure loss Δ*p* in the presence of acoustic perturbations *p*' and *u*'.

$$\frac{\delta\phi}{\phi} \sim \frac{\delta\dot{m}_F}{\dot{m}_F} - \frac{\delta\dot{m}_A}{\dot{m}_A}.$$

The variations in air mass flow rate $\dot{m}_A = \rho_A u A_{Burner}$, where A_{Burner} denotes burner cross sectional area, depend on acoustic perturbations in a straightforward manner

$$\frac{\delta\dot{m}_A}{\dot{m}_A} \sim \frac{\delta u}{u} + \frac{1}{\gamma}\frac{\delta p}{p}.$$

Here the relation $p = \rho^\gamma$, valid for isentropic changes of an ideal gas, has been used to eliminate density from the equation.

Given the assumptions made concering the fuel injection system, variations in fuel mass flow rate $\dot{m}_F = \rho_F u_F A_{Nozzle} \sim \sqrt{\Delta p \rho}$ are controlled by changes in the nozzle outlet pressure:

$$\frac{\delta\dot{m}_F}{\dot{m}_F} \sim -\frac{1}{2}\frac{\delta p}{\Delta p}.$$

Note that it is the pressure drop Δp that appears in the denominator here, not the operating pressure. For a 'soft' fuel injector with small pressure drop, even small changes in acoustic pressure can lead to large variations in fuel mass flow rate.

Combining the relations gathered so far, we obtain the following relation for changes in equivalence ratio due to acoustic perturbations in the vicinity of the injector nozzle:

$$\frac{\delta\phi}{\phi} \sim -\frac{1}{2}\frac{\delta p}{\Delta p} - \frac{\delta u}{u} - \frac{1}{\gamma}\frac{\delta p}{p}. \quad \text{(B3)}$$

Note that these local changes in fuel concentration do not affect the heat release rate immediately, but only after they have been convected with mean flow velocity *u* across the distance $x_j - x_i$ from the injector to the flame. Using the Riemann invariants as introduced above, and defining a convective time flag from fuel injection to flame

$$\Delta t = \frac{x_f - x_i}{u_1}. \quad \text{(B4)}$$

where x_f, x_i are coordinates of the flame front and fuel injection plane, respectively, the closure relation Eq. (21) is obtained. The coefficients F, G in Eq. (21) can be derived by straightforward algebra from (7), (8) and (B3).