MODELLING ASPECTS OF TWO PHASE FLOW

F. MAYINGER, Dr.-Ing.
Institut für Verfahrenstechnik, T.U. Hannover, 6FR

SYNOPSIS In two phase flow scaling is much more limited to very narrowly defined physical phenomena than in single phase fluids. For complex and combined phenomena it can be achieved not by using dimensionless numbers alone but in addition a detailed mathematical description of the physical problem - usually in the form of a computer program - must be available. An important role plays the scaling of the thermodynamic data of the modelling fluid. From a literature survey and from own scaling experiments the conclusion can be drawn that Freon is a quite suitable modelling fluid for scaling steam - water mixtures. However, without a theoretical description of the phenomena nondimensional numbers for scaling two phase flow must be handled very carefully.

INTRODUCTION

The transfer of experimental results, obtained under scaled conditions, to the reality in an industrial plant is an old problem of engineering. In single phase hydrodynamics and heat transfer we have been accustomed to use scaling and modelling laws for many years. Thermohydraulic occurrences are equivalent, if the velocity field, the temperature field and the pressure field are similar. These fields are described by the well-known conservation laws for mass, energy and momentum from which can be derived dimensionless numbers like the Reynolds-, the Froude-, the Euler- or the Nusselt-number and which have to be identical in the test and in the original conditions.

Even for the simpler case of single phase flow it is not possible to satisfy scaling laws completely, e.g. in mixed convection either the forced flow can be scaled with the Reynolds-number or the buoyant induced one can be scaled. Such restrictions become more severe - as we will see - in two phase flows.

Dimensionless numbers for scaling can be derived from:
- differential equations completely describing the physical phenomenon,
- by arranging a matrix from the exponents of the influencing parameters and their dimensions
- and finally simply by trial and error putting together the variables to dimensionless groups.

By these methods or sometimes just by intuitive experience several dimensionless numbers were found for two phase flow and for thermohydraulic processes with phase change already several years ago. Table 1 gives some examples of them. They are usually restricted to a single effect or a simple phenomenon, e.g. the Weber-number to droplet formation and the Buoyancy number to free convection. An exception is the Martinelli-number X, which was found by Martinelli studying two phase pressure drop, but which seems to have a much more general validity.

Whereas in single phase flow only the velocity and the temperature field have to be considered, in two phase the density distribution - i.e. the void fraction \( \varepsilon \) or the quality \( x \) - also has to be taken into account.

In a discussion of modelling aspects one of the first questions is, what has to be scaled in two phase flow. In single phase systems usually the size of an apparatus must be scaled, i.e. the experiments are done with small scaled down test objects. In two phase flow certainly there is the question of the proportion too, but a much more severe problem arises from the large heat input, which is needed for two phase flow and boiling experiments. A geometrical scaling down is usually not possible, because the characteristic, fluidodynamic, volumetric units - e.g. bubbles - are almost independent of the dimensions of the apparatus. Therefore the main emphasis in modelling two phase flow is to scale the original fluid - usually water - with a modelling fluid, which eases the experimental problems because of lower and more convenient parameters of pressure and temperature and needing less energy input.

THERMODYNAMIC SCALING OF THE FLUID

Compared to single phase flow the second phase brings with it much more complication for modelling aspects, especially with respect to the thermodynamic properties of both phases, which have to be similar in the original and in the modelling fluid. Also the phase change behaviour must be comparable. The very first criterion for selecting a modelling fluid, however, is that its thermodynamic properties should be known for the modelling fluid as well as for the original fluid. Besides water the thermodynamic properties of the refrigerants (R11, R12, R13) are well researched and precisely known. In addition these fluids offer convenient experimental conditions. Therefore they are usually used as modelling fluids, especially to scale water.
The first aim for achieving similarity of the thermodynamic conditions is to ensure that the important influencing properties are comparable. This usually can be completely done only for one property and very often the thermodynamic state is selected in such a way that the density ratio \( \rho_1/\rho_2 \) between liquid and vapour is the same in the modelling and in the original fluid. This can be a very serious restriction when other properties are as important as the density.

If there was a common equation of state for both fluids, one could make use of the thermodynamic theorem of corresponding states. One of the simplest equations of state is the Van der Waals equation, which reduced with the critical data, has the form

\[
\left( \frac{p}{p_c} + \frac{3}{v_c} \right) \left( \frac{3}{v_c} - 1 \right) = 8 \frac{T}{T_c}
\]

(1)

and contains only universal constants not restricted to a certain vapour. Certainly the Van der Waals equation allows only a very rough and unprecise prediction of the thermodynamic properties but it gives the hint, that the critical data \( p_c, T_c \) and \( v_c \) may be useful to get a corresponding state for comparable thermal, calorific and even transport properties (ref. 1).

In a single component system the two phases can coexist only along the saturation line, which means, that one thermodynamic property fixes the whole state. So using the critical properties one can either scale with the critical pressure ratio \( p/p_c \)

\[
\left( \frac{p}{p_{c_0}} \right) = \left( \frac{p}{p_{c_0}} \right)_{R12}
\]

(2)

or with any other reduced value of temperature, density, viscosity and so on. Comparisons have proved that the pressure ratio is a useful parameter for scaling between water and refrigerants. As shown in fig. 1, there is only a deviation of a few percent in the density ratio \( \rho_1/\rho_2 \) of liquid and vapor for water and R12 if scaled with the critical pressure ratio, which means that in this case scaling with \( p/p_c \) and with \( \rho_1/\rho_2 \) gives almost the same results.

In many hydrodynamic and heat transfer problems the density ratio may play the most important role but there are other problems like bubble formation, flow pattern, flashing and so on, where other properties like surface tension, viscosity, thermal conductivity are the governing thermodynamic values. A comparison of some properties scaled with the critical pressure ratio is given in fig. 2, where data for viscosity, thermal conductivity and surface tension for water and R12 are compared.

The absolute values of these data differ considerably but the R12 data can easily be transduced to water conditions by simply multiplying with an individual factor as demonstrated in the figure with the dotted lines. This factor then has to be taken into account in the fluid dynamic scaling, e.g. the viscosity factor in the Reynolds-number

\[
Re = \frac{\rho d}{\mu} ; \quad Re_0 = Re_m ; \quad \frac{\mu_m}{\mu_o} = k \cdot \rho_0 \cdot \frac{\mu_m}{\mu_0} = k \cdot \rho_0 \cdot \frac{\mu_m}{\mu_0}
\]

(3)

where according to fig. 2 one gets \( k_2 = 2.15 \) and \( k_1 = 0.8 \). In two phase flow even for this simplification problem on the problem has two correction factors, one for the liquid and another one for the gaseous phase and therefore one has to decide which phase is the more important one governing the problem. For example scaling mass flow rate in two phase flow one can primarily only use the viscosity either of the liquid or of the gas. For the ratio of liquid and vapor mass flow rates an additional condition exists, namely the quality \( x \) or the void fraction. That means, one has to neglect the viscosity influence of one of the phases. From this example already it can be seen that for two phase scaling one has to subdivide into two groups of parameters, namely primary and secondary ones.

In transient conditions, e.g. with flashing in a sea desalination plant or during a loss of coolant accident in water cooled nuclear reactors one must also take into account the first derivatives with respect to pressure of some of the thermodynamic properties. As demonstrated by Belda (ref. 2) and Baker (ref. 3) these derivatives also can be made coincident by simple multiplying factors. Fig. 2 contains examples of two of the derived properties which play an important role in fluid dynamic behaviour during a loss of coolant accident.

**MODELLING OF SIMPLE AND SEPARATE FLUID DYNAMIC EFFECTS**

Under simple circumstances modelling can be successfully achieved using simple dimensionless numbers as for the well known applications of pressure drop and heat transfer in single phase flow. For more complicated cases the dimensionless numbers will give only approximate hydrodynamic and thermodynamic conditions for running the tests and the transfer to the original conditions has to be made by a detailed mathematical description of the physical phenomena.

A simple and separate phenomenon in two phase flow, e.g. is the formation of flow pattern, i.e. the distribution of the two phases in the cross section of a channel. Quandt (ref. 4) deduced predictions for flow pattern from the assumption that the forces - i.e. surface tension, buoyancy and pressure drop - acting on the fluid must be responsible for the phase distribution. From these considerations one can derive dimensionless numbers as e.g. the Weber-number and a dimensionless group, which is the product of the Laplace-constant and a characteristic length, e.g. the bubble diameter.

\[
N_{\text{annular flow}} = \frac{\rho \cdot \frac{w^2 \cdot d}{g}}{\sigma} \quad N_{\text{bubble flow}} = \frac{\sigma}{\rho \cdot g \cdot d^2}
\]

(4)

However, the method of Quandt was not very successful in predicting the regimes of flow pattern. For vertical flow the flow regime map by Bennett (ref. 5) is often used. Zettlemann (ref. 6) demonstrated that simply by modifying the abscissa of the Bennett-diagram with the critical pressure ratio, the flow regime map can be generalized for different fluids and pressures as shown in fig. 3.
The picture is less clear in the case of the void fraction $\varepsilon$ and the slip ratio $s = w/w_i$ which are closely connected with the quality $\xi$ by

$$s = \frac{1 - \varepsilon}{\varepsilon} \frac{\xi}{1 - \xi} \frac{\rho_i}{\rho_g} \tag{5}$$

The slip ratio and through it the void fraction are influenced by the buoyancy forces and by the pressure drop in the channel, so empirical correlations for predicting void fraction usually contain the Froude-number or the Martellini-parameter $X$. But correlations with these dimensionless numbers do not claim general validity for a variety of fluids. A promising step for generalizing the prediction of void fraction was done by Nabizadaeh (ref. 7). Starting from the equation of Zuber and Findlay (ref. 8)

$$\varepsilon = \frac{\xi}{\rho_g} \left[ C_0 \left( \frac{\xi}{\rho_i} + 1 - \frac{\rho_g}{\rho_i} \right) \frac{1}{18} \frac{F}{R} \left( \frac{\rho_g}{\rho_i} \right)^2 \right]^{1/3} \tag{6}$$

and supported by a large number of reliable measurements he developed an empirical correlation for the $C_0$ factor in Zuber-Findlay’s equation

$$C_0 = \left( 1 + \frac{1 - \frac{\rho_g}{\rho_i}}{\frac{\rho_g}{\rho_i}} \right)^{-1} \left[ 1 + \frac{1}{n} Fr^{-1} \left( \frac{\rho_g}{\rho_i} \right)^{1/n} \left( \frac{1 - \frac{\rho_g}{\rho_i}}{\frac{\rho_g}{\rho_i}} \right)^{1/n} \right] \tag{7}$$

$$n = \sqrt{\frac{0.5}{\frac{\rho_i}{\rho_g}}} \tag{8}$$

Applying his correction he gets quite good agreement between correlated and measured data for H$_2$O, D$_2$O, R12 and R13 as demonstrated in fig. 4. This is already an example, in which dimensionless numbers alone are not sufficient for modelling two phase flow.

A very early first step for modelling pressure drop was done by Martinelli (ref. 9). He gave a simple correlation between the two phase multiplier $\Phi$ and the parameter $X$ (see Table 1).

$$\left( \frac{\Delta P}{\Delta H} \right)_{2ph} \Phi = \Phi^2 \left( \frac{\Delta P}{\Delta H} \right)_{1\text{ only}} \Phi = f_1 (X) \tag{9}$$

$$\left( \frac{\Delta P}{\Delta H} \right)_{2ph} \Phi = \Phi^2 \left( \frac{\Delta P}{\Delta H} \right)_{g \text{ only}} \Phi = f_2 (X) \tag{9}$$

In a detailed experimental and theoretical analysis Friedel (ref. 10) starting from Baroczy’s (ref. 11) deliberations elaborated scaling rules for two phase pressure drop. He demonstrated that the two phase multiplier (equation 9) is identical in water and in R12, if the following dimensionless groups are equal under test and under original conditions.

The group with the ratio of the Froude- and the Reynolds-number contains a scaling factor $k$ which is a slight function of the pressure and varies between 1.68 and 1.75. With good approximation a mean value of 1.7 can be taken for $k$ in a pressure range of $50 < p_{H_2O} < 90$ bar and $6 < p_{R12} < 10.8$ bar.

$$\left( \frac{\eta_1}{\eta_2} \right)_{2ph} \left( \frac{\eta_1}{\eta_2} \right)_{1\text{ only}} \left( \frac{\eta_1}{\eta_2} \right)_{g \text{ only}} \tag{10}$$

$$\frac{Fr}{Re^{3/2}} \left( \frac{Fr}{Re^{3/2}} \right)_{1\text{ only}} \frac{Fr}{Re^{3/2}} \left( \frac{Fr}{Re^{3/2}} \right)_{g \text{ only}} \tag{11}$$

Friedel adapts the thermodynamic state by choosing the same viscosity/density-ratio in the model and in the original. Then the mass flow rate in the test is fixed by the scaling factor $k$ via the Reynolds-number of the liquid. Proceeding so, measured data in water and in R12 fall along the same line as demonstrated in fig. 5, where the two phase multiplier is plotted versus the quality $\xi$.

**MODELLING COMBINED EFFECTS**

The fluid dynamic behaviour in an engineering apparatus like a boiler, a nuclear reactor or distillation and extraction columns is affected by a number of phenomena very strongly combined together and usually fluid flow and heat transfer are linked together. There is no general rule how to treat these problems and therefore a few examples - namely burnout behaviour and the blowdown conditions during a loss of coolant accident in a nuclear reactor - shall be discussed.

There are several proposals in the literature for scaling critical heat flux from measurements with refrigerants to water. All start from the simple thermodynamic condition that the density ratio of liquid and vapor has to be the same in the modelling and in the original fluid. Stevens (ref. 12, 13) preferred a more empirical procedure and made a direct comparison of the burnout data in R12 and water with simple scaling factors. He postulated that in addition to the same density ratio and to the identical geometry the quality, or the subcooling at the inlet and the outlet of the heated channel have to be the same in the model and in the original. Therefore only the mass flow rate remains as a free variable to be adjusted in the modelled conditions.

Barnett (ref. 14) used dimensionless analysis for deriving burnout scaling laws. He listed separately all those parameters, thought to mainly govern the physical behaviour of the phenomenon and those, which were thought to have only minor influence to provide primary and secondary groups.

$$\frac{\dot{q}}{h_{g,5}^1 \rho_g} = f \left( \frac{1}{\lambda_i} , \frac{d \cdot \rho_l \cdot h_{g,5}^1 \cdot c_l}{\lambda_i} , \frac{\dot{m}}{\rho_g \cdot h_{g,5}^1} , \frac{\Delta h}{\lambda_i} , \frac{\dot{p}_l}{\rho_g} , \frac{\rho_g \cdot \rho_l}{\lambda_i} \right) \tag{12}$$

From these groups Barnett developed several scaling factors $F$ depending which parameters one chooses as primary ones. Examples of scaling
factors, elaborated and derived by Barnett, are shown in table 2.

Bouré (ref. 15) clearly stated that a comprehensive scaling of the critical heat flux is not possible and that only a restricted similarity of the boiling crisis phenomenon can exist. He distinguishes – similar to Barnett – two groups of parameters, primary and secondary parameters and secondary ones. The latter are accomodated by correction factors determined experimentally and applied to the primary parameters. Doing so, he obtained the following four dimensionless groups

\[
N_1 = K_1 \frac{f}{\rho g} \quad N_2 = K_2 \frac{\Delta h_{nl}}{h_{lg}} \\
N_3 = K_3 \frac{q}{h_{lg} \cdot m \cdot d} \quad N_4 = K_4 \frac{m}{\rho g Y_{lg} \cdot d}
\]

Ahmad (ref. 16) selected three dimensionless groups and arranged them in the modelling parameter \( \Psi \):

\[
\Psi = \left( \frac{n_2 \cdot d}{\rho_1 \cdot d} \right)^{0.67} \left( \frac{m \cdot d}{\eta_1} \right)^{0.33} \left( \frac{m \cdot d}{\eta_2} \right)^{-0.62}
\]

He expressed the critical heat flux \( q \) in the form of the modified boiling number as a function of the modelling parameter

\[
N_{B0}^* = \frac{d}{m \cdot h_{lg}} = f(\Psi, \frac{\Delta h_{nl}}{h_{lg}, \rho g, \frac{1}{d}})
\]

using identical geometrical, density and quality conditions in the test and in the original.

Hein and Kastner (ref. 17) compared measurements in water and in R12 and scaled them according to Bouré as shown in fig. 6. They found that there is an influence of the mass flow rate on the factor \( K_4 \) (see eq. 13), which can be taken into account by a simple mass flow dependent correcting function as demonstrated in fig. 6.

The influence of the mass flow rate becomes greater with non uniform heat flux distribution in the heated channel, mainly, for rod clusters with different power input to the individual rods. This is due to a mixing interaction between the subchannels which are formed by the different rod arrays. This exchange of mass and energy is not taken into account in the scaling and is another example of the limitation to achieve complete similarity in two phase flow.

The thermodynamic behaviour becomes even more complicated if transient conditions are considered, e.g. a blowdown during a loss of coolant accident in a reactor. However, blowdown tests done with R12 (ref. 18) and compared with similar measurements performed with water (ref. 19) gave the promising result that scaling is possible if the thermodynamic conditions are carefully adapted. The blowdown occurrence is too complex to overcome the scaling problems simply by using dimensionless numbers. One has to develop a physical conception of the thermodynamic events and formulate it mathematically in the form of differential equations. Emphasis has to be given to imitate the very severe flashing conditions and the resulting acceleration of the fluid, which can be guaranteed by adapting the same local and terminal pressure ratio. In addition the critical (sonic) mass flow rate and different heat storage conditions have to be scaled. All these problems may only be overcome by a detailed theoretical analysis used as a basis for scaling. This analysis has to predict the results of R12-tests and the water tests as well.

There are two thermohydraulic aspects of interest during blowdown, one is the dryout-delay-time, i.e. that time until dryout occurs, and the other is the heat transfer in the post dryout condition. Starting from the constitution equations for mass, energy and momentum, using equations for the phase velocities derived from pressure drop deliberations and taking into account Hewitt's investigations (ref. 20) into the entrainment behaviour, Belda (ref. 2) developed an analysis for modelling dryout-delay-time which results in a differential integral equation, describing the film thickness as a function of time and place.

\[
\frac{1}{\rho_1} \left( \frac{d}{dt} \left( \frac{M_1(t)}{t} - \frac{M_1(0,t)}{t} \right) + \frac{M_E(t)}{t} \right) = \frac{A_{11}}{\rho_1} \left( \frac{d}{dt} \right) + A_{12} \left( \frac{d}{dt} \right)
\]

\[
\frac{1}{\rho_2} \left( \frac{d}{dt} \left( \frac{\delta g_2(t)}{\delta t} + \frac{\delta h_1(t)}{\delta t} \right) + \frac{\delta g_2(t)}{\delta t} + \frac{\delta h_1(t)}{\delta t} \right) = 0
\]

In equation 16 the local and temporal values of mass flow rate and quality in the rod cluster have to be known from blowdown calculations, which can be performed e.g. with the well-known computer codes like RELAP 4 or BRUCH. For modelling use one can derive from equation 16, the conservation laws and the phase velocities, dimensionless numbers as listed in table 3.

By scaling the hydrodynamic and thermodynamic conditions in such a way quite good agreement between water and R12 behaviour was found as can be seen from fig. 7, where the temporal course of system pressure, mass flow rate, void fraction and pressure drop in the heated section – fuel element or test channel – are plotted for water and for R12 versus the blowdown time.

This conformity, however, does not mean, that the dryout-delay-time is the same in R12 and in water, because only the hydrodynamic aspect of the process is analogous and all the thermodynamic details could not be scaled completely. However, dryout-delay-time can be obtained by calculations from equation 16, which as figure 8 demonstrates, gives good agreement between measured and predicted values in R12. The comparison was done for different locations of the break – in the hot and in the cold leg of the primary system – for a variety of break areas and for different heat flux densities at the moment before the accident occurs.

Also comparisons with water experimental data at
hand show good agreement with the theory as can be seen from fig. 9. This encourages to draw the conclusion that modelling on the detour of a theoretical analysis is a promising procedure for combined and complicated two phase flow phenomena.

To support this a comparison between heat transfer values measured in water and FREON during blowdown was shown in fig. 10, where the scaling was done via the well-known Nusselt-number and a Reynolds-number modified with the quality \( \delta \)

\[
\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}_{g}^{0.4} \quad \text{Re} = \frac{\dot{m}d}{\nu} \left[ \frac{\rho}{\rho_{l}} (1 - \delta) \right]
\]

(07)

The basic assumption in scaling this post-dryout heat transfer phenomenon is that only the vapour is cooling the heated walls directly. The liquid in form of droplets is lowering the temperature in the boundary layer and so having only an indirect effect on the heat transfer. In fig. 10 measurements done with water (ref. 19) and R12 (ref. 21) are compared with correlations from the literature. Keeping in mind that there are many uncertainties arising from the measuring technique the agreement is satisfactory.

CONCLUSIONS

There are certainly a lot of other complicated two phase flow problems, which are of great technical interest. One has to mention for example the forced and free convection behaviour in a boiler, the flow and flashing in sea water desalination chambers as well as the mixing and de-mixing effects in fluid machinery components. Using dimensionless numbers for scaling one very carefully has to separate the primary ones determining the system behaviour and the secondary ones which are of only minor influence.

In not too complicated cases scaling can be achieved by empirical correlations, determined from the most important physical laws, governing the process. For many technical applications however, one is forced to very detailed representation of the thermohydraulic phenomena using computer codes.

This makes it necessary to get a better understanding of the thermohydraulic behaviour in two phase flow systems. A better theoretical approach would also help to generalize experimental data and make fuller use of the numerous experimental results in two phase flow, which up to now have not been correlated. So scaling is not only an expedient to save the expense of experimental research work it also helps in the comparison and the generalization of measured data, For the elaboration of modelling laws for two phase flow more effort needs to be put into the theoretical analysis of the processes.

NOMENCLATURE

- A: area, cross section
- C: specific heat
- Co: distribution parameter
- d: diameter
- F: scaling factor
- F_{max}: maximum deviation
- g: acceleration due to gravity
- h: enthalpy
- h_{lg}: heat of vaporization
- k: scaling factor
- l: length
- \dot{m}: mass flow rate
- \dot{m}_{g}: mass velocity
- n: exponent
- P: primeter
- \Delta P: pressure
- \Delta P_{g}: pressure drop
- Q: heat flux
- S: slip ratio
- T: time
- V: absolute temperature
- \nu: specific volume = 1/\rho
- W: velocity
- X, Y, Z: length coordinates
- X: dimensionless parameter, see fig. 3

GREEK SYMBOLS

- \alpha: heat transfer coefficient
- \beta: see table 2
- \gamma: void fraction
- \epsilon: friction coefficient
- \eta: dynamic viscosity
- \lambda: temperature
- \nu: heat conductivity
- \phi: viscosity = \eta/\rho
- \sigma: density
- \psi: surface tension
- \phi: two phase multiplier
- \psi: modelling parameter

DIMENSIONLESS GROUPS

- Fr: Froude-number
- Gr: Grashof-number
- N: bubble-number
- N_{Bo}: boiling-number
- N_{Nu}: Nusselt-number
- \text{Pr}: Prandtl-number
- \text{Re}: Reynolds-number

SUBSCRIPTS

- B: break
- b: bubble
- C: critical
- CHF: critical heat flux
- d: detachment
- ex: exit
- g: gas
- h: heated
- in: inlet
- l: liquid
- ev: evaporation
- max: maximum
- M: model
- o: original
- s: at steady state condition
- sub: subcooling
- tot: total
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<td>Subcooling number:</td>
<td>( N_{b, s} = \frac{\Delta h_{b, s} \cdot (\rho_1 - \rho_2)}{\varepsilon \cdot \rho_2} )</td>
<td>Phase change number:</td>
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<td>Slip ratio</td>
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<td>Martinelli parameter:</td>
<td>( X = \left( \frac{\rho_0}{\rho_1} \right)^{0.5} \left( \frac{\eta_1}{\eta_2} \right)^{0.5} \left( \frac{1 - \alpha}{\alpha} \right)^{0.9} )</td>
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Table 1: Examples for dimensionless numbers in two-phase systems
\[ \frac{\dot{q}}{f (l, d, \dot{m}, p, \Delta h, h_{g}, p_{l}, p_{g}, c_{l}, \lambda_{l}, \lambda_{g}, Y) \frac{d (P_{g}/p_{s})}{dp_{s}} } \]

\[ \gamma = \frac{d (P_{g}/p_{s})}{dp_{s}} \quad \beta = \frac{d \dot{s}}{dp_{s}} \]

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<th>$F_{d_l}$</th>
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<td>11.72</td>
</tr>
<tr>
<td>12</td>
<td>$h_{g}, p_{l}, p_{g}, c_{g}, \lambda_{g}, \beta$</td>
<td>$\frac{d \cdot \gamma^{0.5} \cdot c_{g}}{h_{g}^{0.5} \cdot \sqrt{p_{l}}} = f \left( \frac{1}{d}, \frac{d \cdot c_{g} \cdot h_{g}^{0.5} \cdot \sqrt{p_{l}}}{\lambda_{g} \cdot \gamma^{0.5}}, \frac{\dot{m} \cdot \gamma^{0.5} \cdot c_{g}}{\sqrt{h_{g}^{0.5} \lambda_{g} \cdot \gamma^{0.5}}}, \frac{p_{l}}{p_{g}}, \frac{\Delta h}{h_{g}} \right)$</td>
<td>23.06</td>
<td>0.317</td>
<td>1.968</td>
<td>11.72</td>
</tr>
</tbody>
</table>

Table 2: Scaling factors for critical heat flux according to Barnett (14)

heat flux

\[ K_{1} = \frac{\dot{q} (2.1) \cdot P_{h}}{m_{\text{tot}} \cdot A_{\text{tot}} \cdot h_{g}} \]

pressure

\[ K_{21} = \frac{P_{h} \cdot \dot{q} \cdot \rho_{h}}{m_{\text{tot}} \cdot d_{p} \cdot \rho_{p}} \]

\[ K_{22} = \frac{P_{g} \cdot \dot{q} \cdot \rho_{g}}{h_{g} \cdot d_{p} \cdot \rho_{p}} \]

\[ K_{24} = \frac{P_{g} \cdot \dot{q} \cdot \rho_{g}}{h_{g} \cdot d_{p} \cdot \rho_{p}} \]

acceleration

\[ K_{3} = \frac{\dot{p} \cdot P_{l}}{m_{\text{tot}} \cdot d_{p}} \]

mass flow rate

\[ K_{4} = \frac{\dot{m} \cdot P_{l}}{m_{\text{tot}} \cdot P_{l}} \]

geometry

\[ K_{5} = \frac{\theta_{l} \cdot d}{\eta_{l}} \]

Table 3: Dimensionless groups for dryout delay scaling during blowdown (Belda)

![Density ratio $\rho_{l}/\rho_{g}$ of water and R12 as function of reduced pressure](https://example.com/fig1.png)

Fig. 1: Density ratio $\rho_{l}/\rho_{g}$ of water and R12 as function of reduced pressure
Fig. 2: Comparison of thermodynamic, transport and derived properties (water and R12)
Fig. 3: Bennett's flow pattern map, modified according to Zetmann (6)

Fig. 4: Void fraction, comparison of water and Freon data, scaled with Zuber-Findlay's equation 8 modified by Nabizadeh (7)
Fig. 5: Two-phase flow pressure drop multiplier scaled according to Friedel (10)

Fig. 6: Mass flow correction for critical heat flux scaling (Bouré, Hein)
Fig. 7: Blowdown during loss of coolant accident, comparison between water tests and scaled R12 data

Fig. 8: Dryout-delay-time, measured (in R12) and calculated data
Fig. 9: Dryout-delay-time, comparison of experimental and theoretical data for water and for R12

Fig. 10: Heat transfer in post dryout conditions during blowdown, comparison of water data and scaled Freon data